**The properties of quadrilaterals**

**Extended investigation Preparation activities**

**Solutions**

**Question 1(a)**

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| Solution    (i)      Solve simultaneous equations  D is the point (*4, 3*)  (ii) *AB*2 = (3-0) 2+ (-4-0) 2= 25 *AB* = 5  *BC*2 = (7-3) 2+ (-1-(-4)) 2= 25 *BC* = 5  *CD*2 = (4-7) 2+ (3-(-1)) 2= 25 *CD* = 5  *DA*2 = (0-3) 2+ (0-(-4)) 2= 25 *DA* = 5  Therefore *ABCD* could be a square. It could also be a rhombus.  **Question 1(a) cont’d**  (iii) Midpoint of *AC* is  Midpoint of *BD* is  The diagonals of a square bisect each other.  (iv)    The diagonals of a square are perpendicular  (v) No, the diagonals of a rhombus also bisect each other at right angles.  If the diagonals are also equal in length, then the quadrilateral is a square. |

**Question 1(b)**

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| Solution    (i) The midpoint of *AC* is (3, -2). Let (*x, y*) represent the coordinates of *B*.  Using the midpoint again  ∴*B*(1, -5).  (ii) Check the lengths of the sides  *AB*2 = (1-0) 2+ (-5-0) 2= 26 *AB* =  *BC*2 = (6-1) 2+ (-4-(-5)) 2= 26 *BC* =  *CD*2 = (5-6) 2+ (1-(-4)) 2= 25 *CD* =  *DA*2 = (0-5) 2+ (0-1) 2= 25 *DA* =  Therefore *ABCD* could be a square. It could also be a rhombus.  Two adjacent sides must be perpendicular also to ensure ABCD is a square.  **Question 1(b) cont’d**    ∴ *ABCD* is a square. |

**Question 1(c)**

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| Solution    *AB*2 = (b-0) 2+ (-a-0) 2= a2+b2  *BC*2 = (a+b-b) 2+ (b-a+a)) 2= a2+b2  *CD*2 = (a-a-b) 2+ (b-b+a) 2= a2+b2  *DA*2 = (0-a) 2+ (0-b) 2= a2+b2  Therefore *ABCD* could be a square. It could also be a rhombus.  Two adjacent sides must be perpendicular also to ensure *ABCD* is a square.    ∴ *ABCD* is a square.  Another way to demonstrate that ABCD is a square is by showing that the diagonals are equal and bisect each other at right angles. |

**Question 2 (a)**

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| Solution  The parallelogram *ABCD* has three known coordinates A(0, 0), B(3, -4) and C(8, -1).    (ii) The properties of the parallelogram:    Check the lengths of the sides  *AB*2 = (3-0) 2+ (-4-0) 2= 25 *AB* = 5  *BC*2 = (8-3) 2+ (-1-(-4)) 2= 34 *BC* =  *CD*2 = (5-8) 2+ (3-(-1)) 2= 25 *CD* = 5  *DA*2 = (5-0) 2+ (3-0) 2= 25 *DA* =  Therefore opposite sides are equal in length.  The parallelogram could also be a rectangle.  **Question 2 (a) (ii) cont’d**  and    Therefore this parallelogram is not a rectangle.  Test the diagonals.  *AC*2 = (8-0) 2+ (-1-0) 2= 65 *AC* =  *BD*2 = (5-3) 2+ (3-(-4)) 2= 53 *BD* =  The diagonals are not equal in length.  Do the diagonals bisect each other?  Midpoint of *AC* is  Midpoint of *BD* is  The diagonals of a parallelogram bisect each other.  Are the diagonals of a parallelogram perpendicular?    The diagonals of a parallelogram are not perpendicular. |

**Question 2 (b)**

(i) Midpoints of diagonals are concurrent. Let (*x, y*) represent coordinates of D

For BD midpoint is 

For AC midpoint is  so *x = c-a* and *y = d – b*

(ii) Gradients of opposite sides are equal.



(ii)

AD2 = BC2 so 

AB2 = DC2 so 



**Question 3**

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| Solution    **KITE**  Two pairs of equal and adjacent sides  Sides not necessarily parallel  One pair of opposite and equal angles  One of the diagonals bisect the other one at right angles  **RHOMBUS**  Two pairs of parallel sides    Four equal sides  Diagonals bisect each other at right angles  Diagonals not equal  **RECTANGLE**    Two pairs of parallel and equal sides  Adjacent sides are perpendicular  Diagonals are equal and bisect each other  Diagonals not necessarily perpendicular    **TRAPEZIUM**  One pair of parallel sides |